TIE-33: Design strength of optical glass and ZERODUR®

1. Introduction

The design strength of a glass part can be an essential functional characteristic when it has to endure stress loads during its application. Often questions arise like

- what is the strength of N-BK 7 ?
- which thickness is necessary for a ZERODUR® disk with diameter 250 mm to endure a pressure of 10 bars?
- will a disk of N-FK 51 develop flaws, when it is subjected to a temperature difference of 30°C?

Such questions lead to the strength of glass directly or indirectly. In the following some general information will be given on the strength of glass. Additionally a calculation procedure will be presented which has been used for a variety of different applications. Data of common optical glass types are enclosed.

2. General aspects of glass strength

The strength of glass and glass-ceramics is not a material property like the Young's modulus e.g. It is dependent on

- the microstructure of the surface which is tension stressed by the load applied,
- the area of the surface exposed to tensile stress,
- the rate of stress increase and
- the environmental media.

A piece of glass breaks when two conditions coincide. The first is the presence of tensile stress at the surface and the second is the presence of a flaw in the region of the tensile stress.

If the flaw is deep (i.e. more than about 0.5 mm as an order of magnitude) or the tensile stress is very high the glass piece is likely to break immediately or in short terms.

When the flaw is small (fairly below 0.5 mm or of microscopic dimensions) the flaw will grow slowly under the influence of the tensile stress. When it reaches a certain depths the growth will accelerate rapidly and breakage will occur. The deepest microflaw within a tensile stress loaded area will determine the strength.

Conservative bending strength values for long term applications of optical glass and ZERODUR® are 8 MPa and 10 MPa respectively. The values are valid for normal surface conditions without scratches and flaws. If higher strength values are required a more detailed analysis has to be made (see below).

2.1 Microstructure condition of the glass surface

The milling and grinding of glass introduces microflaws to the surfaces which extend to depths that lie below the common surface roughness.

The depths of the microflaws depend on the grain sizes of the machining tools and are statistically distributed. Investigations with samples of ZERODUR® and some optical glass types resulted in maximum depths that lie below the maximum grain sizes.

As a rule narrow grain size distributions lead to comparatively narrow strength distributions for ground surfaces. Decreasing grains sizes result in shorter microflaws and in higher strengths consequently.

Polished surfaces have the highest strengths. However, this is valid only if the microcracks introduced by the preceding machining processes have been eliminated. This is achieved by subsequent grinding with decreasing grain sizes each grinding process taking off a material layer at least three to four times as thick as the maximum microcrack depth of the preceding process. The strength distributions of glasses with polished surfaces scatter significantly more than those for glasses with ground surfaces.

2.2 Area of the tensile stress loaded surface

The admissible strength for a tensile stress loaded area decreases with increasing area. Deeper microcracks occur more frequently when the surface area increases.

2.3 Rate of stress increase

The higher the rate of stress increase is the higher is the bending strength of the glass part.

2.4 Environmental media

Environmental media influence the microcrack growth under long term tensile stress loads. Liquid water enhances the crack growth and leads to lower strengths values consequently, whereas glass within vacuum exhibits the highest strength values.

3. Strengthening of glass

When a design stress calculation as described below results in an admissible stress load that is too low for the requirements of the application it is possible to strengthen the glass part by a pre-stressing procedure. This procedure introduces compressive stress in the surface regions of the glass part. A bending load first has to overcome this compressive pre-stress. The design strength therefore is increased by the amount of the pre-stress.

There are some restrictions on the application of that method. It presupposes a significant thermal expansion of the material Therefore ZERODUR® cannot be pre-stressed. The resulting stress birefringence excludes the application of such glass parts in polarization optics and very faint light ray deviations may distort images in high resolution optics.

An alternative method for strengthening glass is etching. This method can also be applied on ZERODUR®.

4. Recommendations to maintain the glass strength

Cleaning procedures should use liquids in abundance and soft clothes. Avoid rubbing. Check the glass part on scratches. Scratches will make the design strength become invalid.

Frames must not exert forces on glass parts, especially point like forces. Avoid direct metal contact. Adhesive joints shall be made using soft glues with layers not too thin, so that they can compensate shear stresses.

5. Calculation procedure

5.1 Mathematical model

The mathematical model which is used to describe the results of strength tests on glass parts is the Weibull distribution with two parameters.

$$F(\sigma) = 1 - \exp(-(\sigma / \sigma_0)^{\lambda}) \qquad (5\text{-}1)$$

with:

$F(\sigma)$ - Probability of failure at bending stress σ

σ_0 - Characteristic strength ($F(\sigma_0) = 63{,}21\ \%$)

λ - Weibull factor (slope of the Weibull straight line and a measure for the scatter of the distribution.)

This distribution function is widely used in product lifetime statistics and allows to derive predictions on the failure rates for collectives of identical parts. Basing on laboratory test results obtained under well defined conditions one can calculate design strengths for loads and conditions posed by special application requirements.

5.2 Outline of the calculation procedure

The calculation procedure described in the following is a simplified excerpt of the procedure that has been published in [6]. The notation used in this document is adopted from this publication.

From the evaluation of strength test results one obtains $\sigma_0(S_L, 63\%, R)$ and λ for a definite glass type and surface condition.

$\sigma_0(S_L, 63\%, R)$ is the characteristic strength for the laboratory test surface area S_L and stress increase rate R.

In the model employed the design strength depends on the parameters:

S_V - the area of the tensile stress loaded surface

R_V - the stress increase rate or

t_V - the stress load duration time (when constant)

F_V - the admissible probability of failure

The design strength a is derived from the laboratory strength by dividing it by f_{FOS}, the so-called factor of safety.

$$s_k = s_0 / f_{FOS} \qquad (5\text{-}2)$$

5.3 Factor of safety

The factor f_{FOS} is factorized once again:

$$f_{FOS} = f_A \cdot f_P \cdot f_F \qquad (5\text{-}3)$$

The formulae for the individual factors are derived on the basis of the Weibull model using the laws on probability.

5.3.1 Area factor f_A

The area factor f_A is calculated according to the following formula

$$f_A = (S_V / S_L)^{1/l} \qquad (5\text{-}4)$$

This formula assumes constant stress within the loaded area. It is a conservative approach since in many cases the tensile stresses have a maximum amount and fall off with increasing distances to that maximum. For a more rigid calculation one has to use $S_{eff,V}$ which is obtained by weighing the maximum stress with the stress distribution function instead of S_V.

5.3.2 Probability factor fp

The formula for the probability factor f_p is

$$f_p = \frac{1}{\left(\ln \frac{1}{1 - F_V}\right)^{1/l}} \qquad (5\text{-}5)$$

5.3.3 Fatigue factor f_F

The general formula for the calculation of the fatigue factor f_F is

$$f_F = \left(\frac{t_{eff,V}}{t_{eff,L}}\right)^{1/n} \qquad (5\text{-}6)$$

where

$t_{eff,V}$ - effective loading time for the application

$t_{eff,L}$ - effective loading time at laboratory

n - environmental stress corrosion constant

In the special case for a stress load constant in time this formula reads explicitly

$$f_F = (t_V \cdot f_A \cdot f_P \cdot R \cdot (n+1)/s_0)^{1/n} \qquad (5\text{-}7)$$

For application cases with time varying loads $t_{eff,V}$ has to be calculated instead of t_V by using a weighing function that describes the load variation with time.

The stress corrosion constant n has been determined for several glass types (see data sheets attached). When no experimental results are available, n may be estimated according to formula

$$n = 38 - 2.6 \cdot a \qquad (5\text{-}8)$$

with: α - coefficient of thermal expansion in units 10^{-6}/K usually a(-30,+70°C)

For the derivations of the formulae and more detailed information about the calculation of the factors f_A, f_P and f_F see the publication of Dr. Exner quoted above.

6. Bending strength of ZERODUR®

The following table and diagrams show measurement results for the characteristic strength, Weibull factor and stress corrosion constant n for ZERODUR®. The test procedure was according to DIN 52292-1 Double ring method R 30-6 with a stress increasing rate of 2 MPa/s, room climate and a test area of 113 mm^2.

Material	**Surface condition**	**Characteristic strength s_0 [MPa]**	**Weibull factor l**	**Stress corrosion constant n / Medium**
ZERODUR®	SiC 600	108.0	16.0	51.7 Air 50% [2]
ZERODUR®	SiC 320	71.3	12.4	59.2 Air 50% [3]
ZERODUR®	SiC 230	57.5	15.7	30.7 Water [3]
ZERODUR®	SiC 100	53.6	18.7	
ZERODUR®	D 15 A	130.6	10.6	(for all
ZERODUR®	D 35	78.7	15.7	surfaces)
ZERODUR®	D 64	64.0	12.5	
ZERODUR®	D 151	53.7	22.7	
ZERODUR®	D 251	48.8	11.1	
ZERODUR®	Opt. polish	293.8	5.3	
ZERODUR®	D 64 etched	219.8	6.0	
ZEROOUR®	SiC 600 at 77K	192.7	10.7	
glassy ZERODUR®	SiC 320	66.6	16.7	
ZERODUR® M	D 64	63.9	12.5	
ZERODUR® M	D 64 etched	291.6	3.8	

Table 1: Characteristic strength, Weibull factor and stress corrosion factor of ZERODUR®

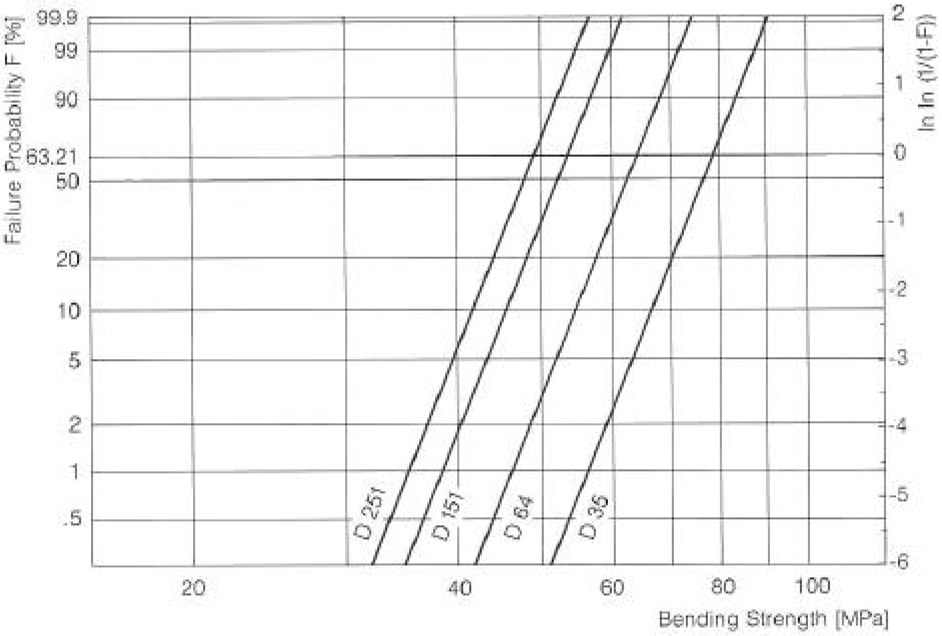

Figure 1: Failure probability of ZERODUR® for test surfaces processed with bonded grit of different sizes.

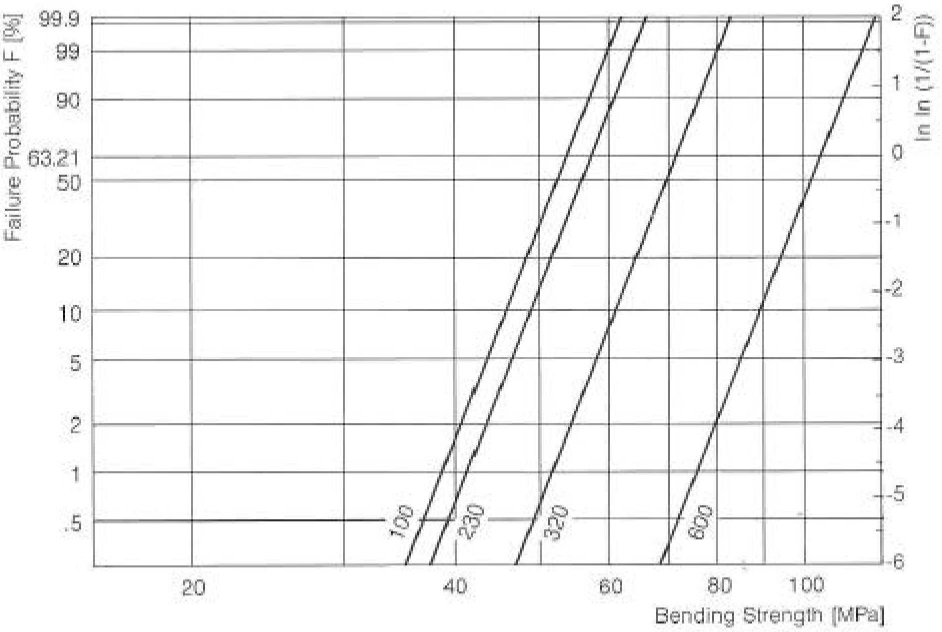

Figure 2: Failure probability of ZERODUR® for test surfaces processed with loose grit of different sizes.

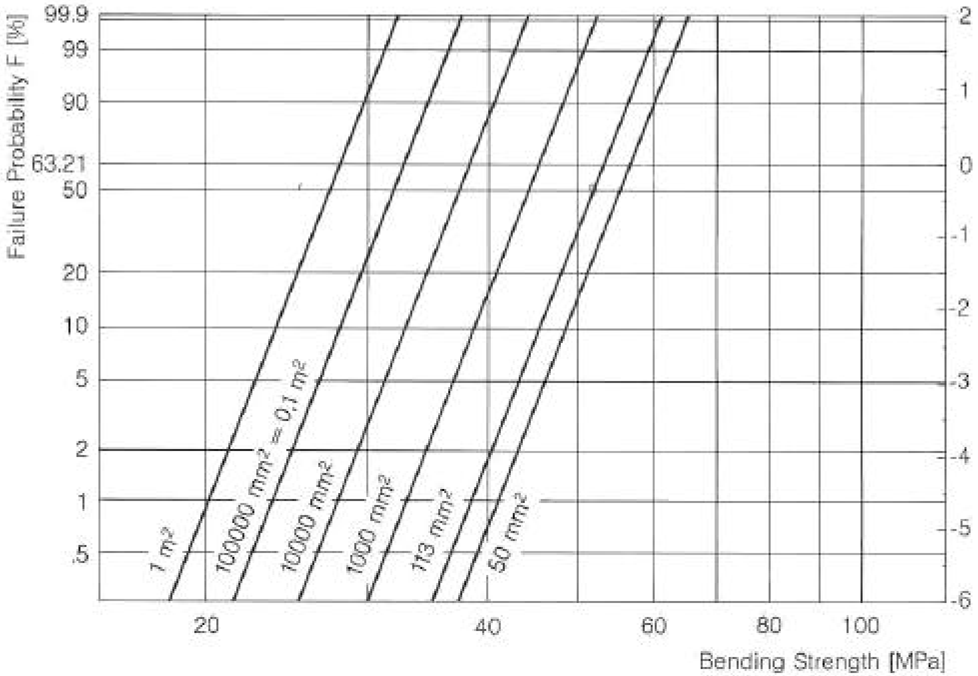

Figure 3: Failure probability F of ZERODUR® as a function of test surface area S_L.

The values for the test surface area $S_L = 113\ mm^2$ in figure 3 are based on measurements (Rate of stress increase : 2 MPa/s. Test surface processed with D 151, air as surrounding medium). The values for the other test surfaces are based on calculations.

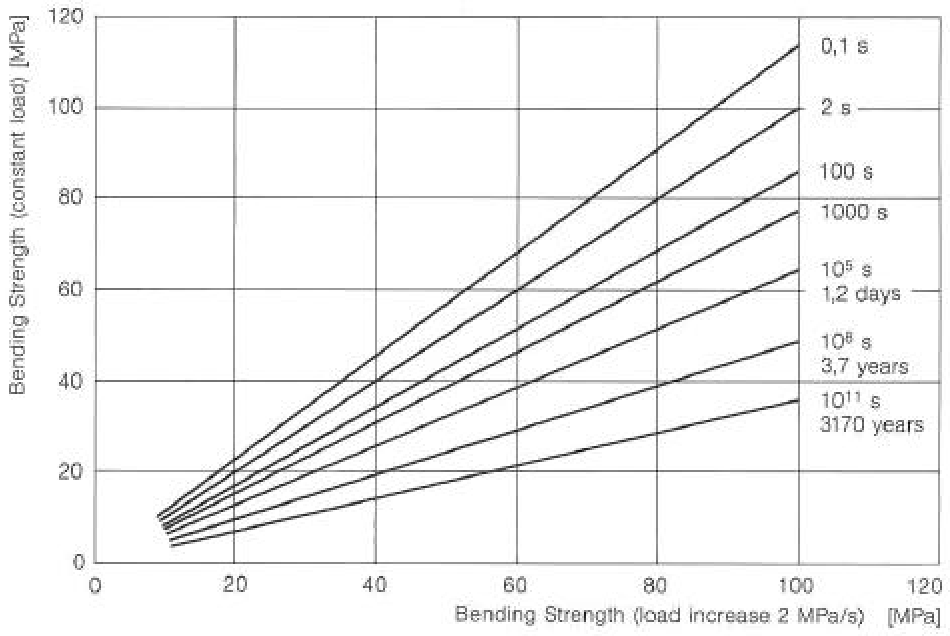

Figure 4: The bending strength of ZERODUR® under constant load.

7. Bending strength of optical glass

The following table shows some measurement results for the characteristic strength, Weibull factor and stress corrosion constant n for optical glass. The test procedure was according to DIN 52292-1 Double ring method R 30-6 with a stress increasing rate of 2 MPa/s, room climate and a test area of 113 mm^2.

Material	**Surface condition**	**Characteristic strength** s_0 **[MPa]**	**Weibull factor** l	**Stress corrosion constant n / Medium**
N-BK 7	SiC 600	70.6	30.4	19.5 Air 50% (*)
N-BK 7	D 64	50.3	13.3	
N-BK 7	D 64 etched	234.7	4.1	
N-ZK 7	SiC 600	68.9	14.1	26.3 Air 50% (*)
N-BaK 1	SiC 600	58.9	8.2	18.2 Air 50% (*)
N-SK 16	SiC 600	62.3	19.3	21.6 Air 50% (*)
N-LaK 8	SiC 600	70.0	29.9	23.4 Air 50% (*)
F 2	SiC 600	57.1	25.0	15.4 Air 50% (*)
N-BaSF 64	SiC 600	70.1	23.9	19.0 Air 50% (*)
N-LaF 21	SiC 600	75.9	28.6	21.9 Air 50% (*)
SF5	SiC 600	55.3	10.6	16.7 Air 50% (*)
SF6	SiC 600	49.2	5.4	16.9 Air 50% (*)
SF 57	SiC 600	39.1	14.7	16.4 Air 50% (*)
KzFS N4	SiC 600	49.4	25.5	26.3 Air 50% (*)
UG 11	SiC 600	63.4	4.4	16.4 Air 50% (*)
KG 3	SiC 600	61.4	11.3	24.2 Air 50% (*)
Floatglas	SiC 600	80.9	26.9	18.1 Air 50% [4]
Duran	SiC 600	75.7	14.9	30.5 Water [5]

(*) calculated according formula $n = 38 - 2.6 * \alpha$

Table 2: Characteristic strength, Weibull factor and stress corrosion factor of optical Glass

8. Fracture toughness of glass

So far the strength of optical glass was considered from a statistical point of view by measuring the bending strength of samples and subsequently estimating the failure probabilities. Looking at a single flaw in a material the maximum bending strength depends on the size of the flaw and geometry in the material. For example in case of a flaw with a short depth in a thick plate with tensile forces acting normal to the crack plane one can define a stress intensity factor K_I by:

$$K_I \approx 2\sigma_0\sqrt{a} \qquad (8\text{-}1)$$

with σ_0 being the nominal stress perpendicular to the stress plane and a the depth of the flaw. A flaw will result in a fracture if :

$$K_I \geq K_{IC} \qquad (8\text{-}2)$$

K_{IC} is the critical stress intensity factor for crack mode I (tensile forces normal to the crack plane, crack propagation perpendicular to the forces). K_{IC} is a material constant. For glasses without additional strengthening the value is typically ≤ 1. Table 3 gives fracture toughness values of some glasses:

Glass	**K_{IC} [MPa $m^{1/2}$]**
N-BK7	1.1
F5	0.9
ZERODUR®	0.9
SF6	0.7

Table 3: Fracture toughness values of some glasses [7,9]

For a given nominal stress the plate will break for a critical depth a_c of

$$a_c \approx \left(\frac{K_{IC}}{2\sigma_0}\right)^2 \qquad (8\text{-}3)$$

Numerical example: For the characteristic strength of ZERODUR® of samples with D64 surface condition $\sigma_0 \approx 64$ MPa (table 1) and ZERODUR® $K_{IC} \approx 0.9$ MPa $m^{1/2}$ the critical flaw size a_c is approx. 49 µm. This flaw size compares to the grain sizes for D64 bonded diamond grains (table 4).

Most glasses exhibit slow crack growth for a stress intensity factor well below the critical value. As mentioned above the most important sub-critical crack growth occurs in the presence of water (amounts less then 10 mg per m^2). The velocity of the crack can be described by [8]:

$$\frac{da}{dt} = A * \left(\frac{K_I(a)}{K_{IC}}\right)^n \qquad (8\text{-}4)$$

a denotes the depth of the crack, A is a constant and n is the stress corrosion constant.

Typical values for the stress corrosion constant can be found in table 1 and 2. Typically the sub critical crack grow can start from 0,25*K_{IC}.

8. Appendix: Grain Sizes

	Designation	**Mean size [µm]**	**Max. size [µm]**	**ASTM equiv.**
Bonded diamond grains **acc. DIN 848**	D251	231	250	60/70
	D151	138	150	100/120
	D107	98	106	140/170
	D64	58	63	230/270
	D35	36	40	--
	D15A	12.5	15	--
Loose silicon-carbide grains **acc. FEPA Std. 42-D-1984**	SiC 100	116	149	--
	SiC 230	53	84	--
	SiC 320	29	49	--
	SiC 600	9	19	--

Table 4: Grain sizes of grinding tools

9. Literature

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[3] M.J.Viens NASA Technical memorandum 4185 Washington 1990
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[9] Viens, Michael J.: “Fracture toughness and crack growth parameters of ZERODUR®”, NASA technical memo A969903 (1990)

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